Micromagnetics for Spintronics

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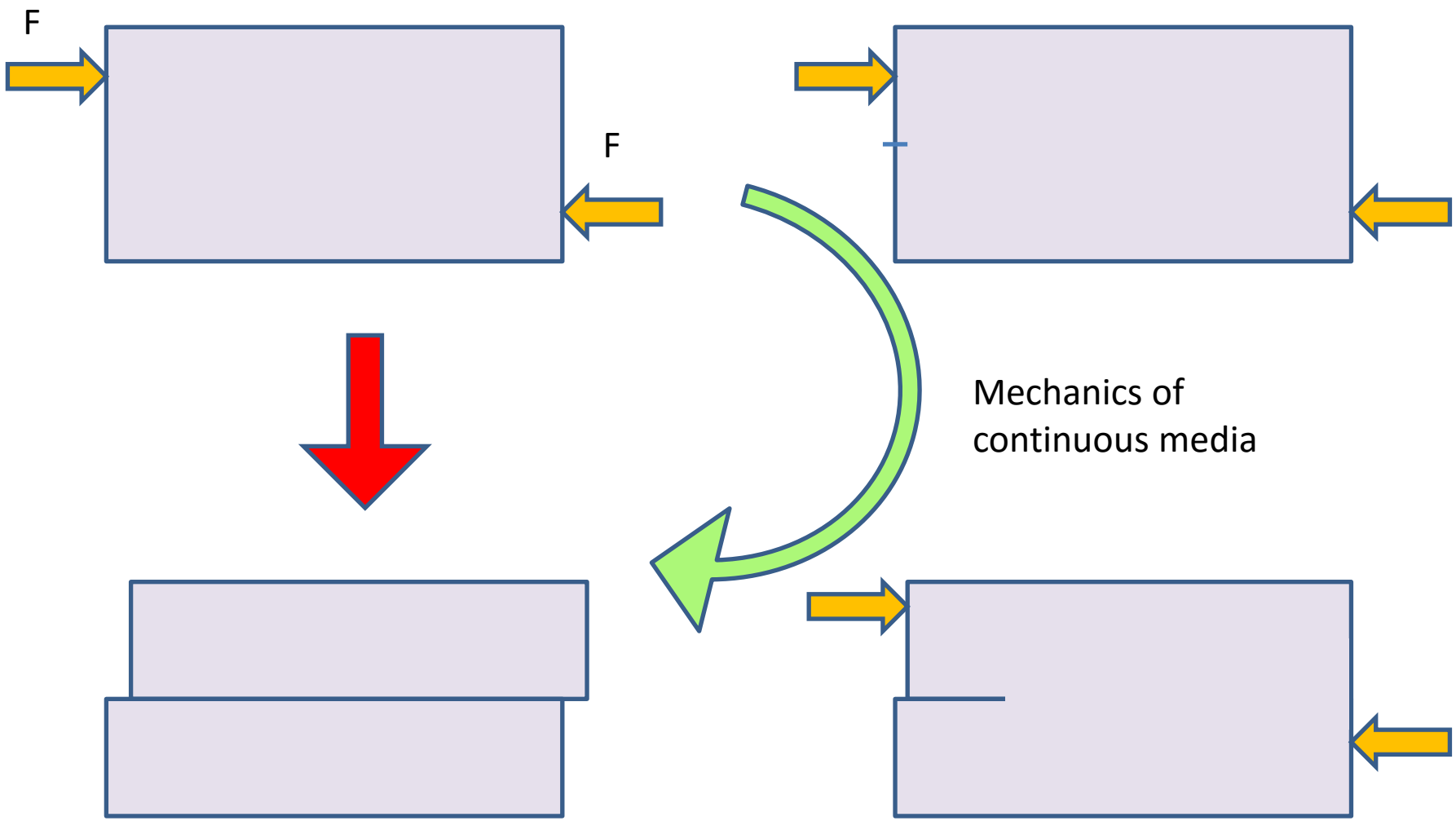
What does Micromagnetics ?



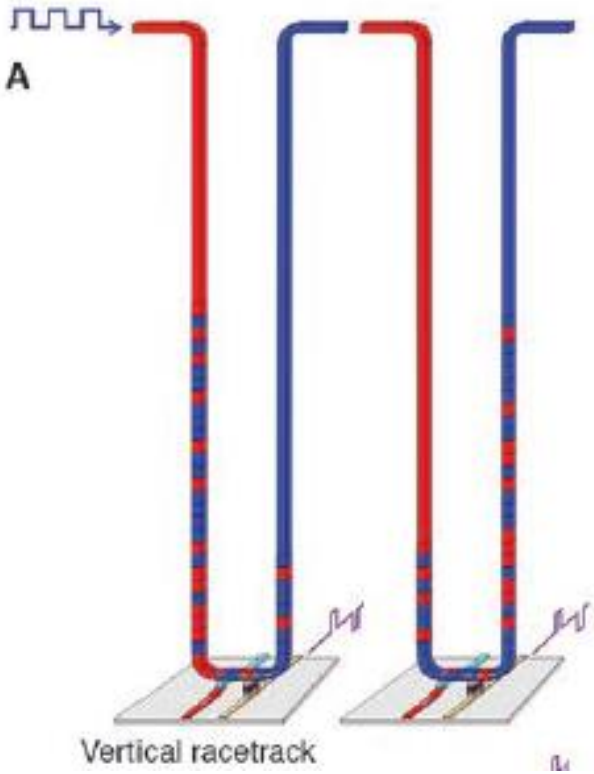
Micromagnetics = mechanics of continuous magnetic structures

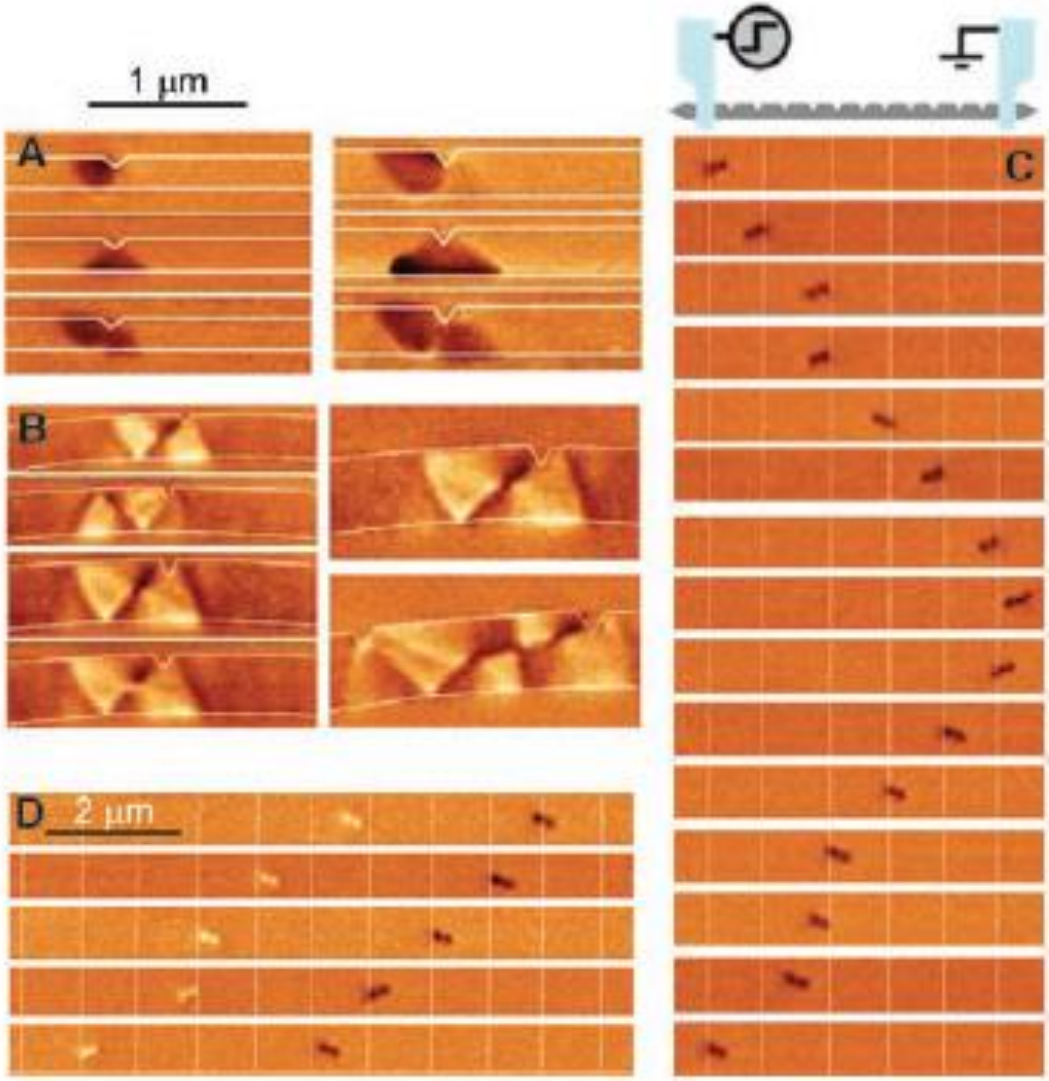
Why use Micromagnetics ?

Magnetic Domain-Wall Racetrack Memory

www.sciencemag.org SCIENCE VOL 320 11 APRIL 2008

Stuart S. P. Parkin,* Masamitsu Hayashi, Luc Thoma





APPLIED PHYSICS LETTERS **98**, 062501 (2011)

Dynamics of two coupled vortices in a spin valve nanopillar excited by spin transfer torque

N. Locatelli,[1] V. V. Naletov,[2,a)] J. Grollier,[1] G. de Loubens,[2] V. Cros,[1,b)] C. Deranlot,[1] C. Ulysse,[3] G. Faini,[3] O. Klein,[2] and A. Fert[1]

[1]*Unité Mixte de Physique CNRS/Thales and Université Paris Sud 11, 1 av. A. Fresnel, 91767 Palaiseau, France*

[2]*Service de Physique de l'Etat Condensé (CNRS URA 2464), CEA Saclay, 91191 Gif-sur-Yvette, France*

[3]*Laboratoire de Photonique et de Nanostructures (LPN), CNRS, Route de Nozay, 91460 Marcoussis, France*

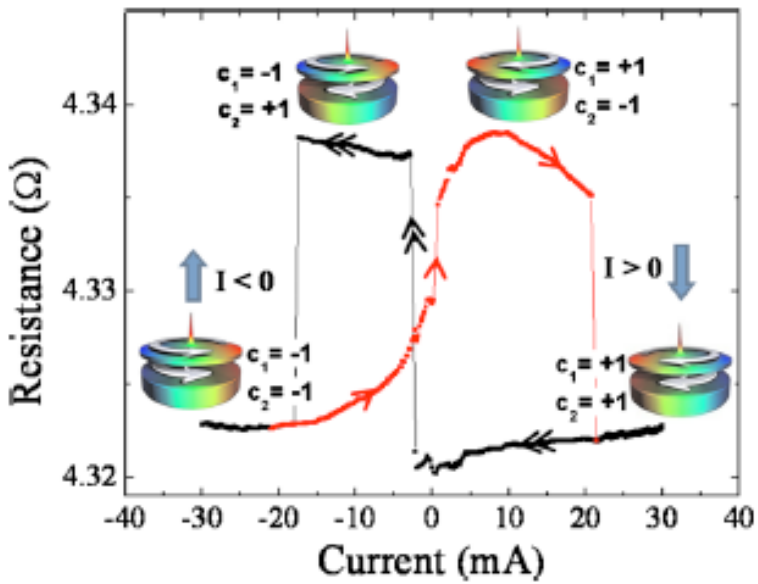
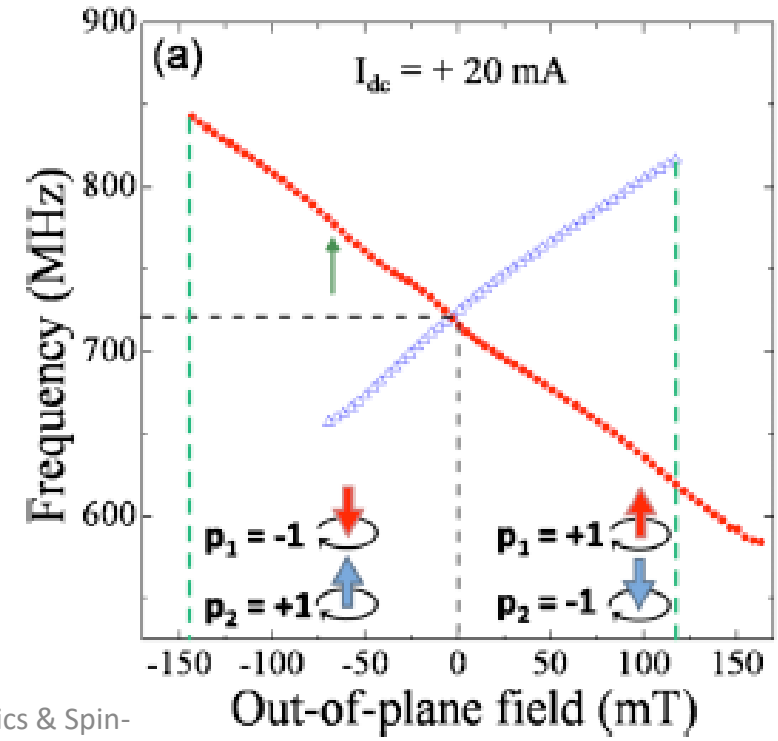


FIG. 1. (Color online) Resistance vs bias current at zero field for a 200 nm diameter Py (15 nm)/Cu (10 nm)/Py (4 nm) nanopillar. A parabolic contribution ($\propto I_{dc}^2$), due to Joule heating, has been subtracted for clarity. The sketches depict the four accessible chirality configurations with c_1 (c_2) being the vortex chirality in the thin (thick) layer.



PRL 109, 096602 (2012) PHYSICAL REVIEW LETTERS week ending 31 AUGUST 2012

Current-Induced Switching of Perpendicularly Magnetized Magnetic Layers Using Spin Torque from the Spin Hall Effect

Luqiao Liu,[1] O. J. Lee,[1] T. J. Gudmundsen,[1] D. C. Ralph,[1,2] and R. A. Buhrman[1]
[1]*Cornell University, Ithaca, New York 14853, USA*
[2]*Kavli Institute at Cornell, Ithaca, New York, 14853*
(Received 24 April 2012; published 29 August 2012)

We show that in a perpendicularly magnetized Pt/Co bilayer the spin-Hall effect (SHE) in Pt can produce a spin torque strong enough to efficiently rotate and switch the Co magnetization. We calculate the phase diagram of switching driven by this torque, finding quantitative agreement with experiments. When optimized, the SHE torque can enable memory and logic devices with similar critical currents and improved reliability compared to conventional spin-torque switching. We suggest that the SHE torque also affects current-driven magnetic domain wall motion in Pt/ferromagnet bilayers.

PHYSICAL REVIEW B 89, 024418 (2014)

Central role of domain wall depinning for perpendicular magnetization switching driven by spin torque from the spin Hall effect

O. J. Lee,[1] L. Q. Liu,[1] C. F. Pai,[1] Y. Li,[1] H. W. Tseng,[1] P. G. Gowtham,[1] J. P. Park,[1] D. C. Ralph,[1,2] and R. A. Buhrman[1]
[1]*Cornell University, Ithaca, New York 14853, USA*
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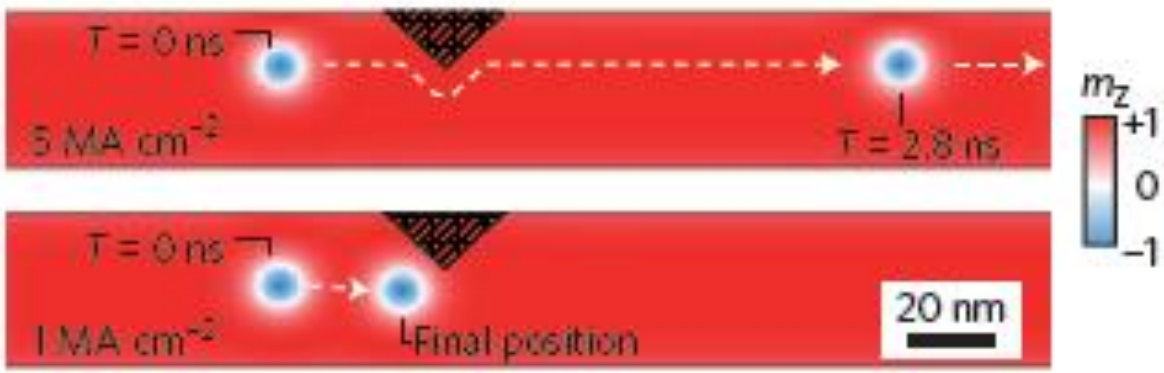
(Received 17 November 2013; published 28 January 2014)

We study deterministic magnetic reversal of a perpendicularly magnetized Co layer in a Co/MgO/Ta nanosquare driven by spin Hall torque from an in-plane current flowing in an underlying Pt layer. The rate-limiting step of the switching process is domain wall (DW) depinning by spin Hall torque via a thermally assisted mechanism that eventually produces full reversal by domain expansion. An in-plane applied magnetic field collinear with the current is required, with the necessary field scale set by the need to overcome DW chirality imposed by the Dzyaloshinskii-Moriya interaction. Once Joule heating is taken into account the switching current density is quantitatively consistent with a spin Hall angle $\theta_{SH} \approx 0.07$ for 4 nm of Pt.

Skyrmions on the track

Albert Fert, Vincent Cros and João Sampaio

Magnetic skyrmions are nanoscale spin configurations that hold promise as information carriers in ultradense memory and logic devices owing to the extremely low spin-polarized currents needed to move them.



IEEE TRANSACTIONS ON MAGNETICS, VOL. 50, NO. 11, NOVEMBER 2014 3401006

Domain Wall Motion Device for Nonvolatile Memory and Logic — Size Dependence of Device Properties

Shunsuke Fukami[1,2], Michihiko Yamanouchi[1,3], Shoji Ikeda[1,2,3], and Hideo Ohno[1,2,3,4]

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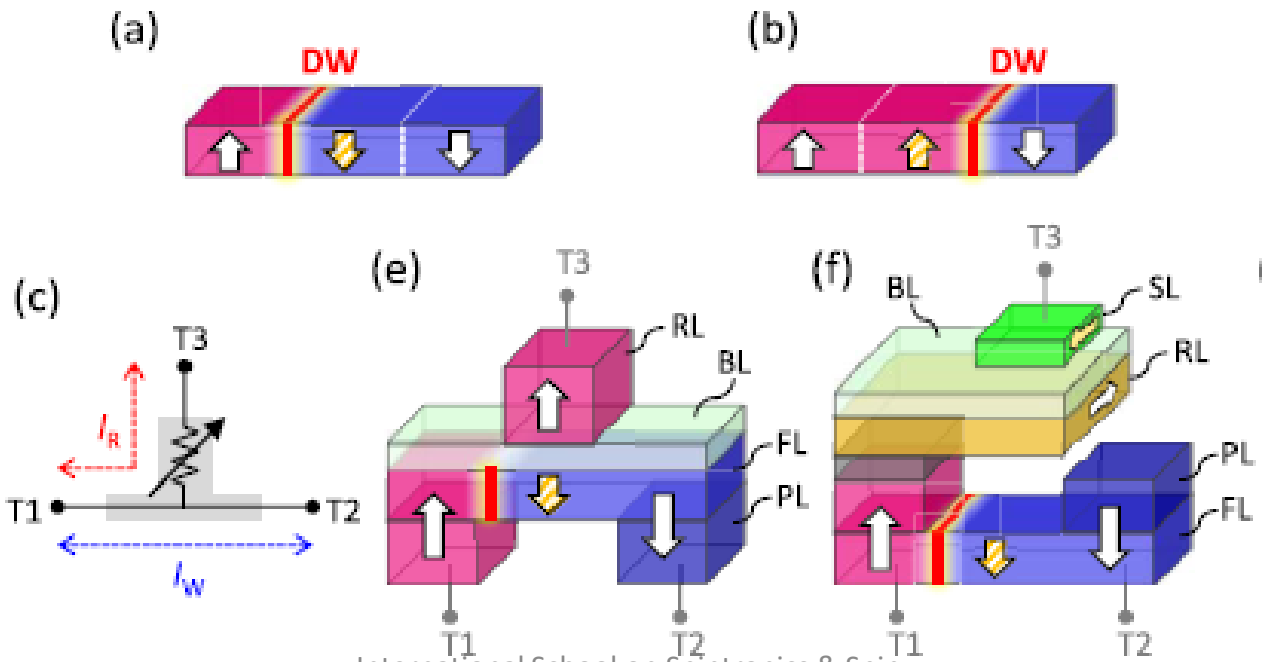
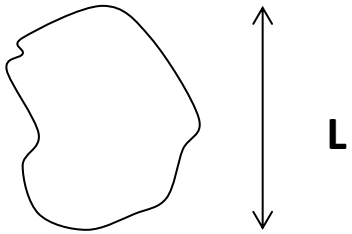
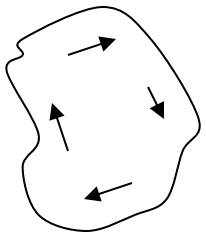
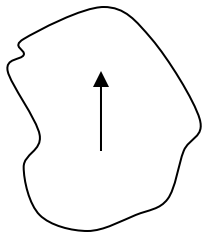


When not to use Micromagnetics ? nm-size objects

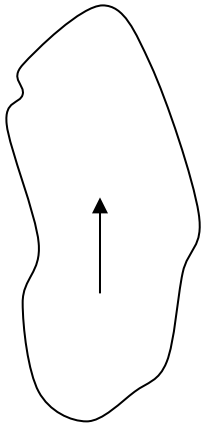






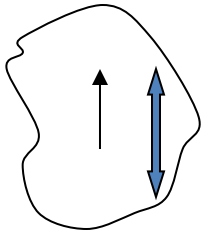
$$E_{ech} \approx A\left(\frac{\pi}{L}\right)^2, E_{dem} \approx 0 \qquad E_{ech} \approx 0, E_{dem} \approx \frac{1}{3}\frac{\mu_0 M_s^2}{2}$$

stable monodomain state for $\frac{1}{3}\frac{\mu_0 M_s^2}{2} < A\left(\frac{\pi}{L}\right)^2 \Leftrightarrow L < \pi\sqrt{3}\Lambda$



Demagnetising factor N

$$L < \pi\frac{\Lambda}{\sqrt{N}}$$



With anisotropy, the transition size increases too

The progressive development of Micromagnetics

1926	P. Weiss	Concept of magnetic domains
1931	F. Bitter	Observation of domain (walls)
1931	K. Sixtus, L. Tonks	Dynamics of domain walls
1932	F. Bloch	Bloch wall
1935	L. Landau, E. Lifchitz	Effective field, LL dynamic equation
1944	L. Néel	Bloch walls for different anisotropies
1946	C. Kittel	Single domain particles
1948	W. Döring	Domain wall dynamic structure, mass
1955	T.L. Gilbert	Gilbert damping, LLG equation
1963	W.F. Brown	*Micromagnetics*
1969	A.E. LaBonte	Asymmetric Bloch wall: numerical micromagnetics

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What is Micromagnetics ?

Basic assumptions of Micromagnetics

1) Fixed magnetization modulus

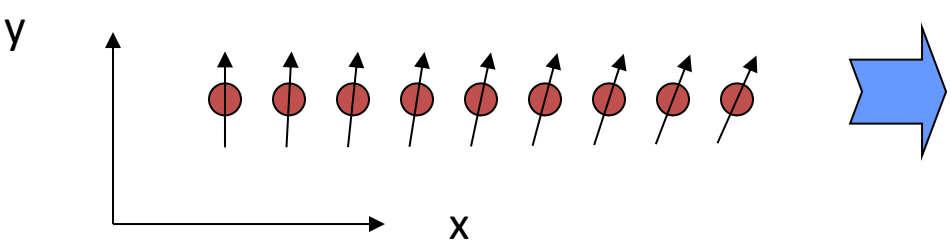
$$\vec{M} = M_s(T)\vec{m} \qquad |\vec{m}| = 1$$

« micromagnetization »

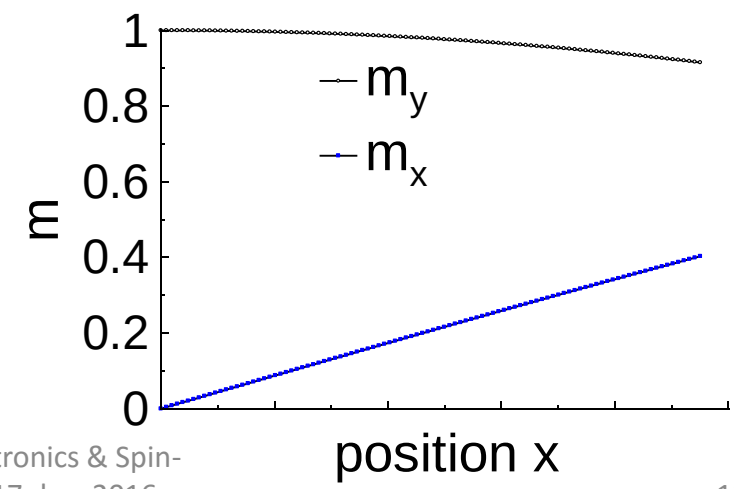
2) Slow variations at the atomic scale -> continuous model

$$\vec{m}(\vec{r}, t)$$

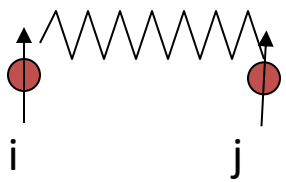
atomic spins



continuous distribution

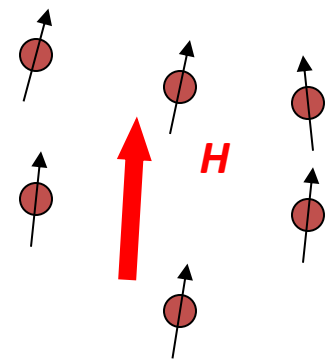


Basic magnetic energy terms

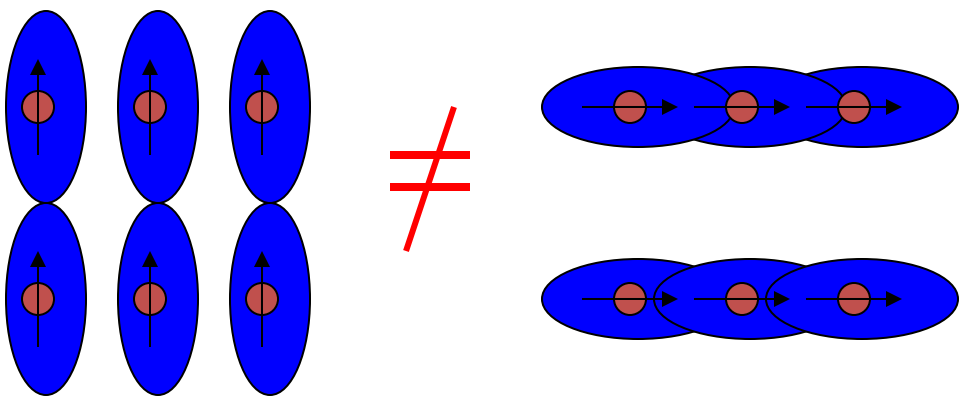


Exchange

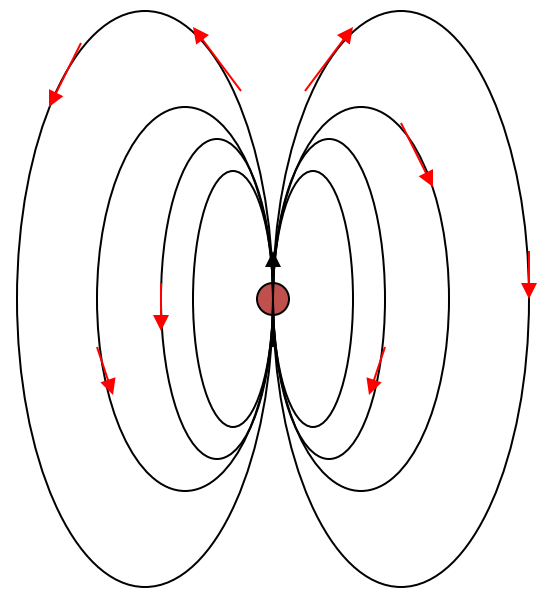
$$E = -J\vec{S}_i \cdot \vec{S}_j$$



Applied field



Anisotropy



Demagnetising field

Micromagnetic equations : statics

$$E = A(\vec{\nabla}\vec{m})^2 + KG(\vec{m}) - \mu_0 M_s \vec{m}\cdot\vec{H} - \frac{1}{2}\mu_0 M_s \vec{m}\cdot\vec{H}_D$$

exchange anisotropy applied field demagnetizing field

Statics : minimise $\int_V E\, d^3r$ → Brown equations

$$\boxed{\vec{H}_{eff} \times \vec{m} = \vec{0}}$$

+ boundary conditions

$$\boxed{\frac{\partial \vec{m}}{\partial \vec{n}} = \vec{0}}$$

effective field

$$\vec{H}_{eff} = \vec{H}_{applied} + \vec{H}_{demag} + \vec{H}_{aniso} + \vec{H}_{exchange}$$

$$\boxed{\vec{H}_{eff} = -\frac{1}{\mu_0 M_s}\frac{\delta E}{\delta \vec{m}}}$$

NB « functional derivative »

$$\frac{2A}{\mu_0 M_s}\Delta\vec{m}$$

Demagnetizing field : magnetostatic approximation

$$\vec{B} = \mu_0 \left(\vec{H} + \vec{M} \right) \quad \text{with} \quad div\,\vec{B} = 0 \quad \text{and} \quad \overrightarrow{rot}\,\vec{H} = \vec{j}$$

demagnetizing field

$$\begin{cases} div\,\vec{H}_D = -div\,\vec{M} \\ \overrightarrow{rot}\,\vec{H}_D = \vec{0} \end{cases}$$

volumic magnetic charges

applied field

$$\begin{cases} div\,\vec{H}_{app} = 0 \\ \overrightarrow{rot}\,\vec{H}_{app} = \vec{j} \end{cases}$$

+ boundary conditions

$$(\vec{H}_D^{ext} - \vec{H}_D^{\text{int}}) \cdot \vec{n} = \vec{M} \cdot \vec{n}$$

surfacic magnetic charges

$$(\vec{H}_D^{ext} - \vec{H}_D^{\text{int}}) \cdot \vec{t} = 0$$

$$\vec{H} = \vec{H}_D + \vec{H}_{app}$$

Basic micromagnetic characteristic lengths

$$\Delta = \sqrt{\frac{A}{K}}$$

Bloch wall width parameter

$A=10^{-11}$ J/m, $K=10^{2} – 10^{5}$ J/m^3

Δ= 1 - 100 nm

$$\Lambda = \sqrt{\frac{2A}{\mu_0 M_s^2}}$$

exchange length

$M_s = 10^6$ A/m

Λ= some nm

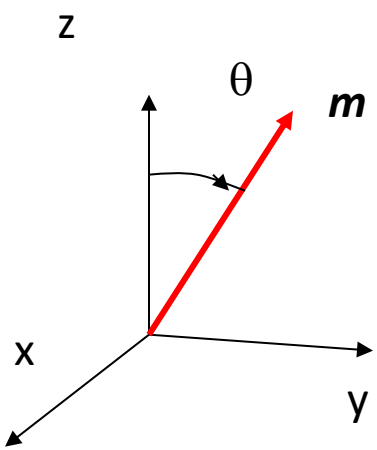
$$Q = \frac{2K}{\mu_0 M_s^2} = \left(\frac{\Lambda}{\Delta}\right)^2$$

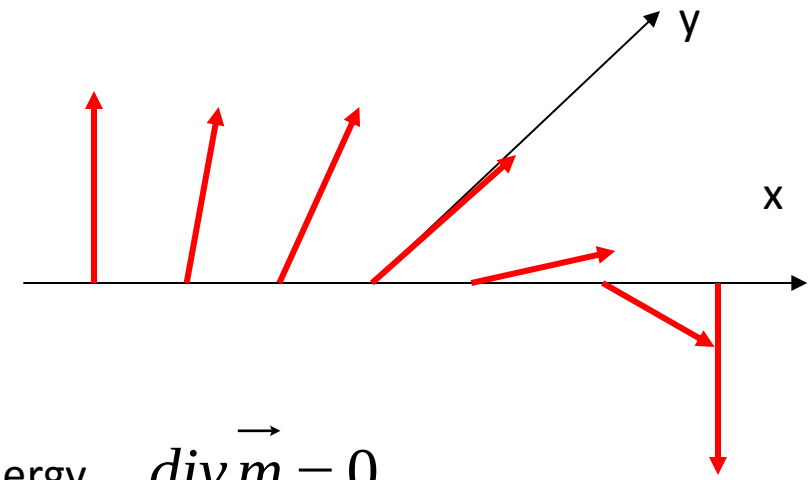
Quality factor

$Q > 1$ hard material

$Q << 1$ soft material

The Bloch wall : length Δ



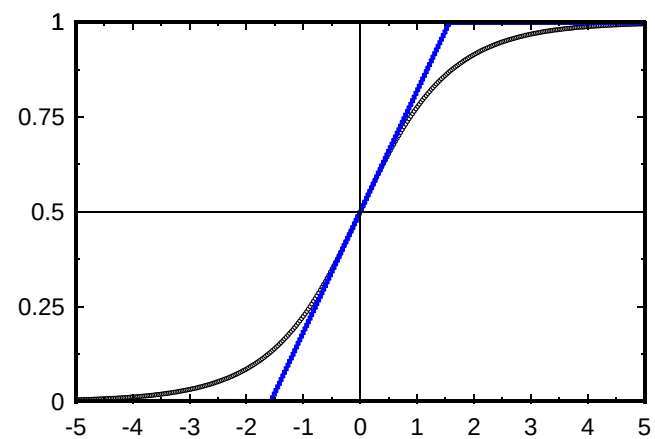


No demag energy $div\,\vec{m} = 0$

$$\vec{m} = \begin{pmatrix} 0 \\ \sin\theta(x) \\ \cos\theta(x) \end{pmatrix}$$

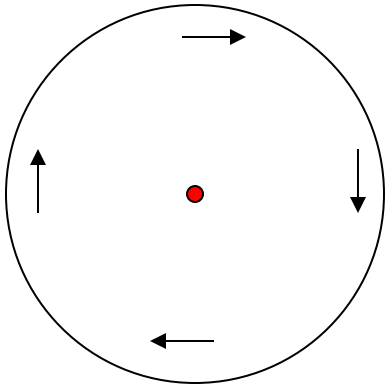
$$E = A\left(\frac{d\theta}{dx}\right)^2 + K\sin^2\theta$$

$$\theta(-\infty) = 0, \qquad \theta(+\infty) = \pi$$



The magnetic vortex : length Λ

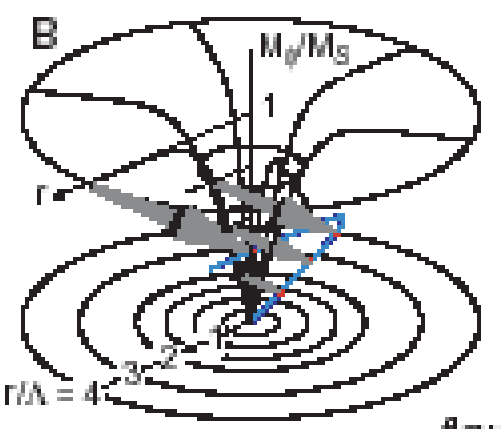
2D magnetization



$$\vec{m} = \begin{pmatrix} y/r \\ -x/r \\ 0 \end{pmatrix} \qquad \begin{cases} div\,\vec{m} = 0 \\ \vec{m}\cdot\vec{n} = 0 \end{cases}$$

Divergence of the exchange energy $E_{ech} = A\left(\vec{\nabla}\vec{m}\right)^2 = A/r^2$

3D magnetization



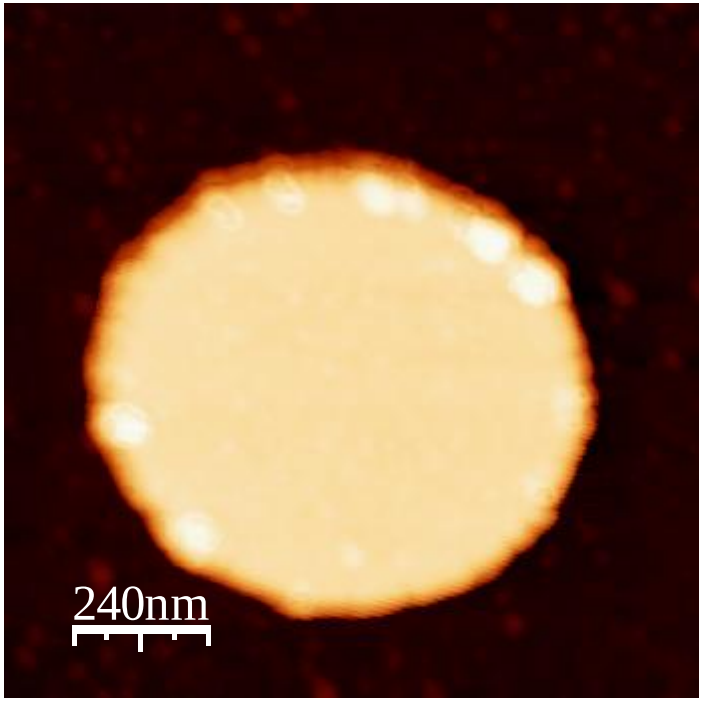
Ansatz

$$\vec{m} = \begin{pmatrix} \sin\theta(r)\ y/r \\ -\sin\theta(r)\ x/r \\ \cos\theta(r) \end{pmatrix} \qquad \begin{cases} div\,\vec{m} = 0 \\ \vec{m}\cdot\vec{n} \neq 0 \end{cases}$$

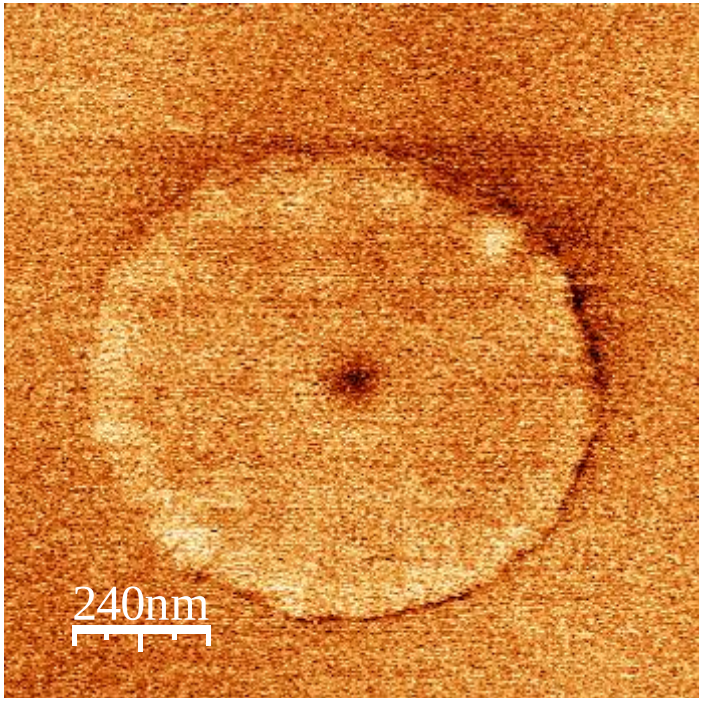
$$E_{ech} = A\left[\sin^2\theta/r^2 + \left(\frac{d\theta}{dr}\right)^2\right]$$

$$E_{dem} = \frac{\mu_0 M_s^2}{2}\cos^2\theta$$

Observation by Magnetic force microscopy (MFM)

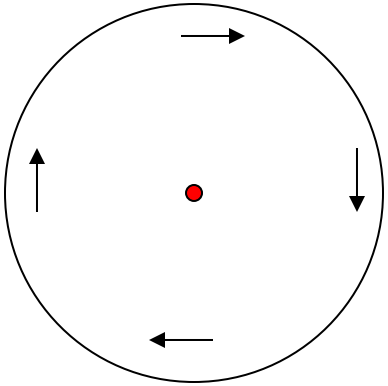


Topography

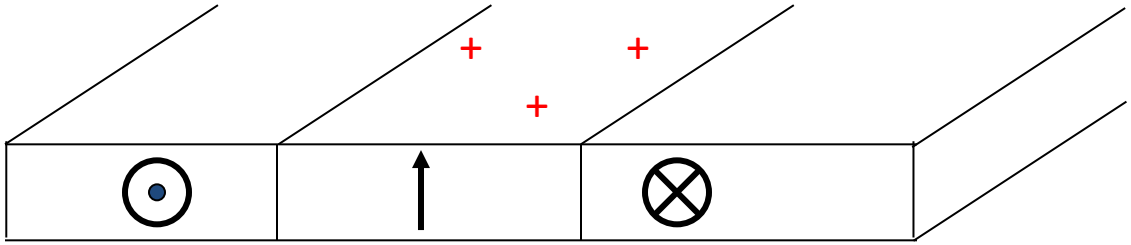


Magnetic image

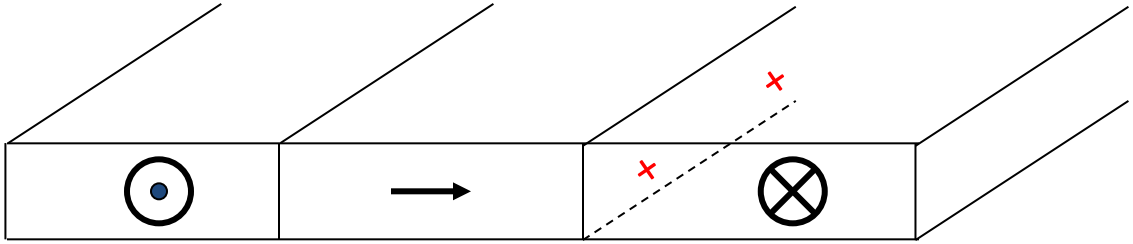
NiFe (50 nm)
T. Okuno
JM Garcia



The Néel wall



Bloch wall

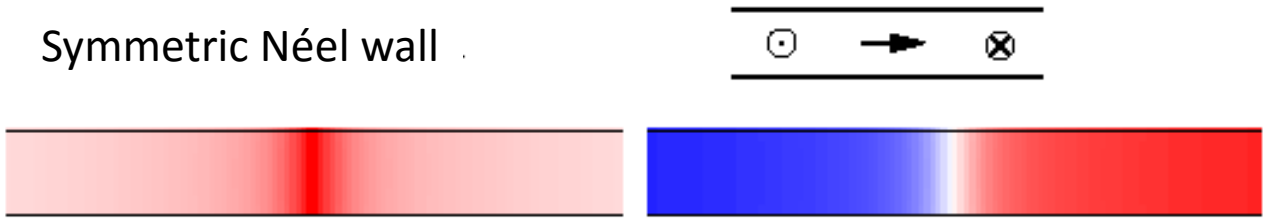


Néel wall

Thin film without anisotropy, or small in-plane anisotropy

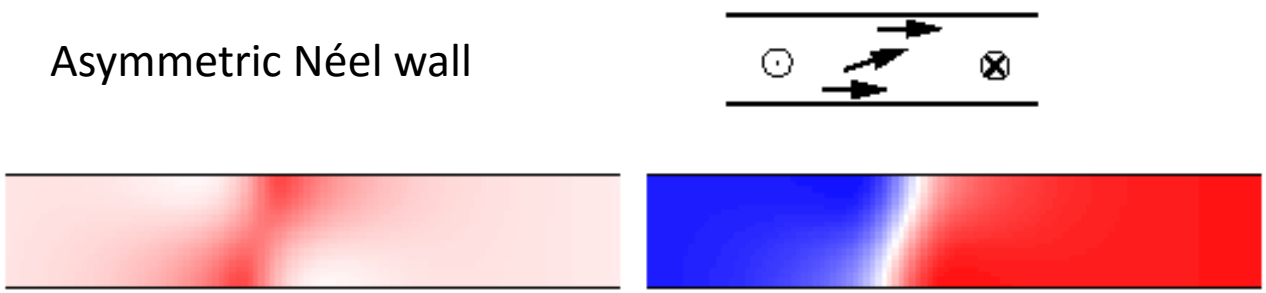
Walls in soft thin films

Symmetric Néel wall



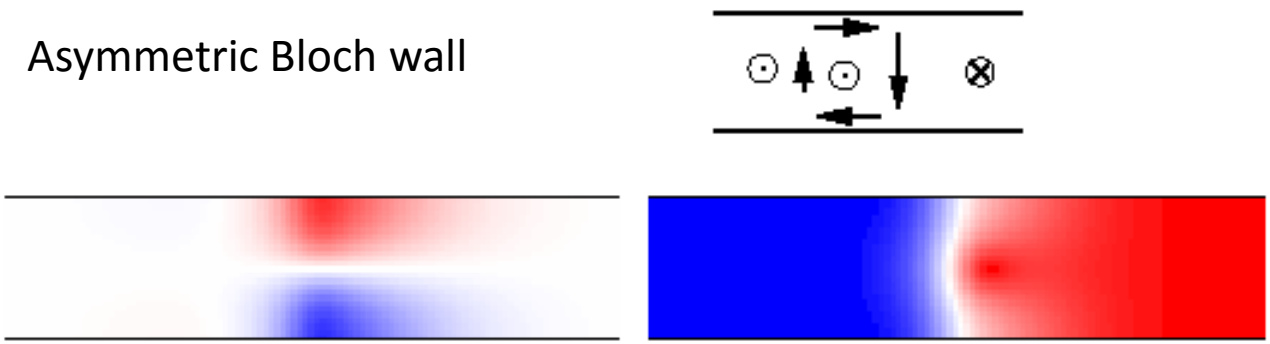
composante → NiFe 30 nm composante ⊙

Asymmetric Néel wall



NiFe 40 nm

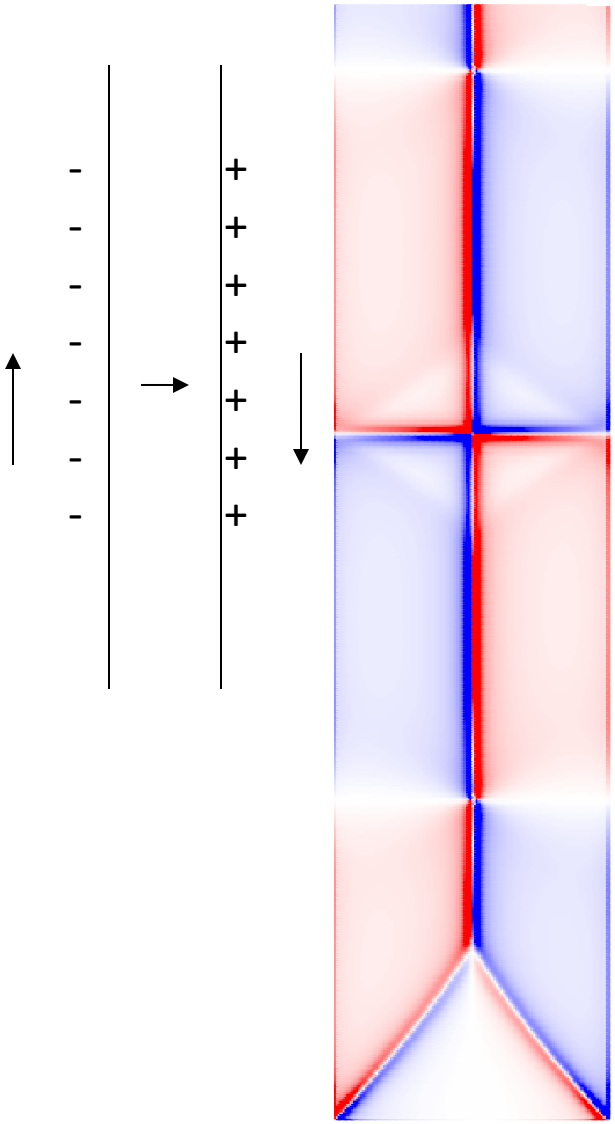
Asymmetric Bloch wall



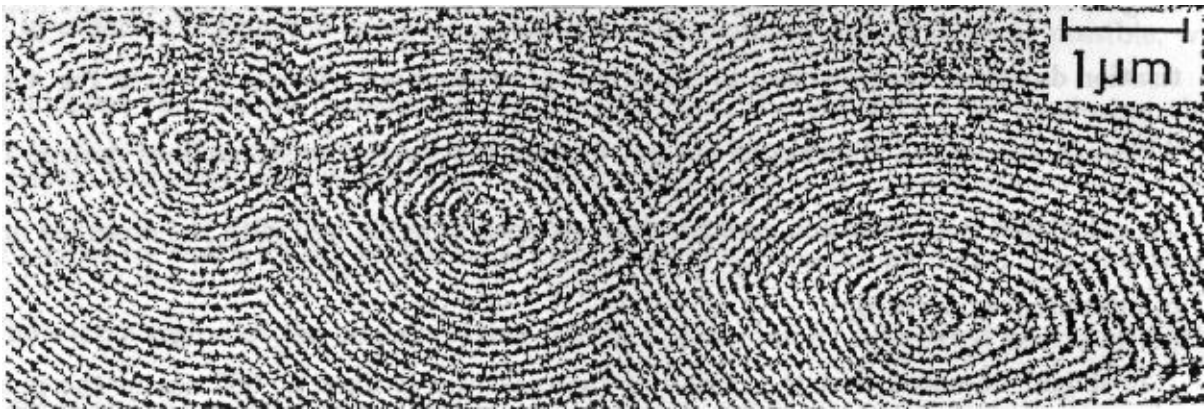
NiFe 50 nm

→ ← ⊙ ⊗

2D instability of the Néel wall : cross-tie



map of the magnetic charges



electron holography image

A. Tonomura et al., Phys. Rev. **B25** 6799 (1982)

Magnetization dynamics

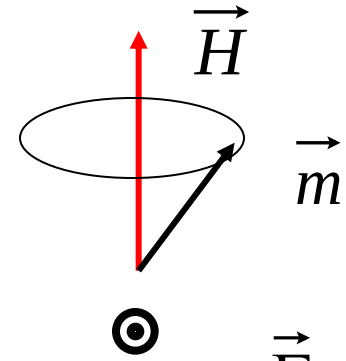
$$\vec{L} = -\vec{M} / \gamma$$

γ gyromagnetic ratio (>0)

$$\gamma = \frac{g\mu_B}{\hbar} = g\,\frac{e}{2m}$$

Angular momentum dynamics

$$\frac{d\vec{L}}{dt} = \vec{\Gamma}$$



$$\vec{\Gamma} = \mu_0 M_s\, \vec{m} \times \vec{H}$$

$$\frac{d\vec{m}}{dt} = \gamma_0\, \vec{H} \times \vec{m}$$

Can be found directly from quantum mechanics

$$\gamma_0 = \mu_0 \gamma \approx 2.2\ 10^5\ S.I.$$

28 GHz/ T

Micromagnetic equations : dynamics

Effective field

$$\vec{H}_{eff} = \vec{H}_{applied} + \vec{H}_{demag} + \vec{H}_{anisotropy} + \vec{H}_{exchange}$$

$$\frac{2A}{\mu_0 M_s}\Delta\vec{m}$$

$$\boxed{\vec{H}_{eff} = -\frac{1}{\mu_0 M_s}\frac{\delta E}{\delta \vec{m}}}$$

$$\boxed{\frac{d\vec{m}}{dt} = \gamma_0 \vec{H}_{eff} \times \vec{m} + \alpha \vec{m} \times \frac{d\vec{m}}{dt}}$$

Landau-Lifshitz-Gilbert (LLG)

α : Gilbert damping parameter

$$= \frac{\gamma_0}{1+\alpha^2}\left[\vec{H}_{eff} \times \vec{m} + \alpha \vec{m} \times \left(\vec{H}_{eff} \times \vec{m}\right)\right]$$

(solved form of LLG)

Properties of the magnetization dynamics

1) $$\frac{d(\vec{m}^2)}{dt} = 2\vec{m}.\frac{d\vec{m}}{dt} = 0$$ Conservation of the magnetization modulus

2) $$\frac{dE}{dt} = -\mu_0 M_s \vec{H}_{eff}.\frac{d\vec{m}}{dt} = -\alpha\mu_0 M_s \vec{H}_{eff}.\left(\vec{m}\ x\ \frac{d\vec{m}}{dt}\right)$$

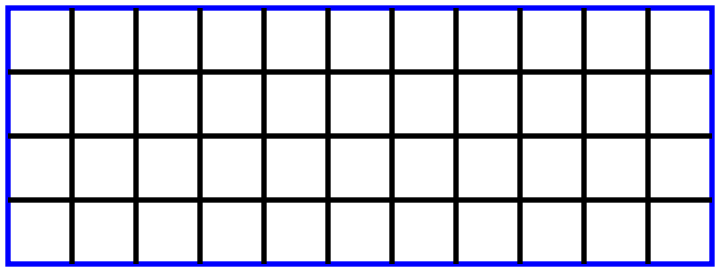
$$= -\alpha\mu_0 M_s \frac{d\vec{m}}{dt}.\left(\vec{H}_{eff}\ x\ \vec{m}\right) = -(\alpha\mu_0 M_s / \gamma)\left(\frac{d\vec{m}}{dt}\right)^2$$

Decrease of the energy with time : the magnetic system is not isolated

Numerical Micromagnetics

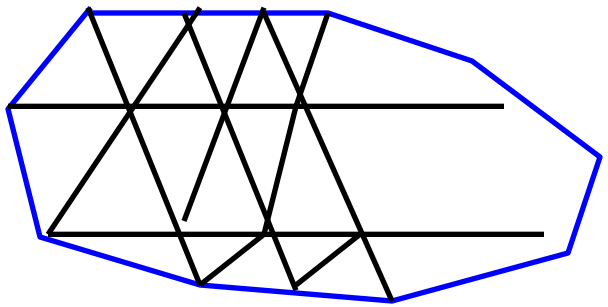
Two types of numerical schemes

Finite differences :



- easy to code
- well adapted to demag field calculation (FFT)
- most used

Finite elements :



- can handle any shape
- can easily implement local mesh refinement

Design rules

Cell size smaller than the characteristic length of the problem (of the structure if statics only)

In principle, results for decreasing size meshes should be extrapolated to zero mesh size to reach the continuous limit

Warnings :
- mesh friction
- mesh orientation effects
- Bloch points
- Brown paradox : role of defects

Time schemes

$$\left(1+\alpha^2\right)\frac{dm}{dt} = -\gamma_0\left[m \times \boldsymbol{H}_{eff}\right] - \alpha\gamma_0 m \times \left[m \times \boldsymbol{H}_{eff}\right]$$

$$\boldsymbol{H}_{eff} = \frac{2A}{\mu_0 M_s}\Delta\boldsymbol{m} + \boldsymbol{H}_A + \boldsymbol{H}_D - \frac{1}{\mu_0 M_s}\frac{\delta\varepsilon_K}{\delta\boldsymbol{m}}$$

Heat diffusion equation :

$$\frac{\partial T}{\partial t} = D\ \Delta T$$

Stability of explicit scheme for dt < $(dx)^2$ / D

⇨ Small time steps, or implicit scheme

Codes

1) OOMMF (http://math.nist.gov/oommf): since 1999
- public and free; finite differences
- easy to use, versatile inputs (scripts)
- requires expertise in C++ for modification

2) MuMax3 (http://mumax.github.io) : since 2011
- public and free; finite differences
- extremely fast (runs on GPUs)

3) Nmag (http://nmag.soton.ac.uk/nmag) : since 2007
- public and free; finite elements

Home-made programs

Commercial codes

Other energy terms and effective fields

Inclusion of some thermal fluctuations : Langevin model

$$\vec{H}_{eff} = -\frac{1}{\mu_0 M_s}\frac{\delta E}{\delta \vec{m}} + \vec{H}_{th} \quad ; \quad \vec{M} = M_s \vec{m}$$

$$\left\langle \vec{H}_{th} \right\rangle = \vec{0} \qquad \left\langle H_{th}^{i}(t) H_{th}^{j}(t') \right\rangle = \mu\, \delta_{ij}\, \delta(t-t')$$

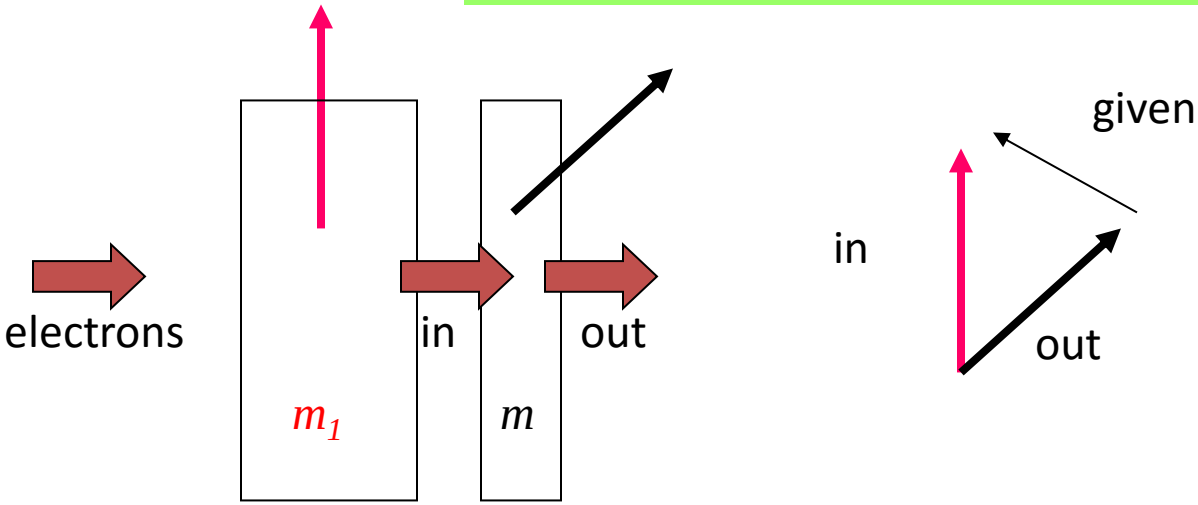
$$\mu = \frac{2k_B T\alpha}{\gamma_0 M_s V} \qquad \sigma(H_{th}^{i}) = \sqrt{\frac{2k_B T\alpha}{\gamma_0 M_s V}}$$

V: volume of a mesh cell

$$\hbar\omega << k_B T$$

W. F. Brown, Phys. Rev. **130** (1963) 1677

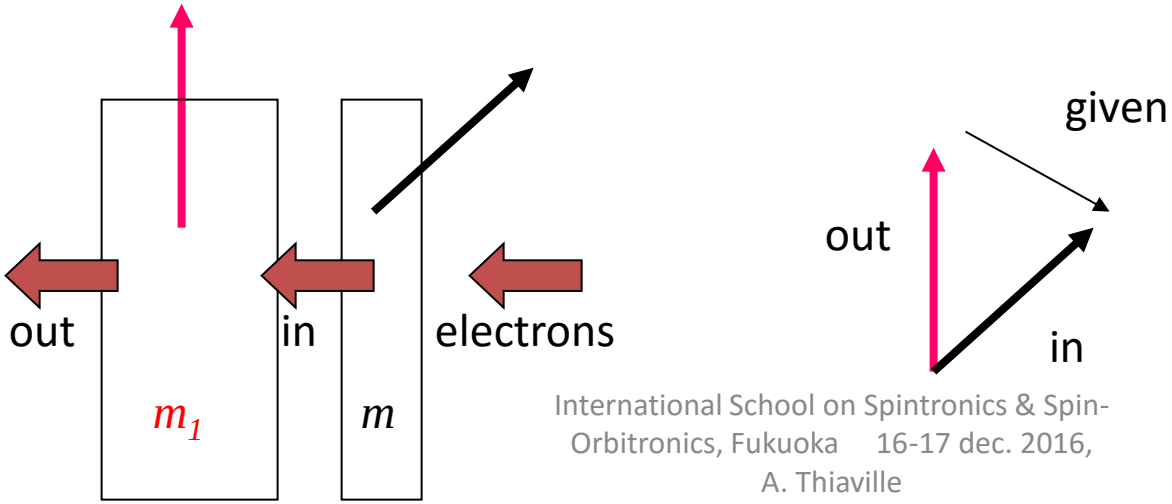
Spin transfer torque (STT)



Favors a parallel alignment of F to F1

$$\left.\frac{d\vec{m}}{dt}\right)_{spin-transfer} = \frac{Jg\mu_B P}{2eM_s D}\left(\vec{m}_1 - \vec{m}\right)_\perp = \frac{1}{\tau}\vec{m}\times\left(\vec{m}_1\times\vec{m}\right)$$

Slonczewski STT

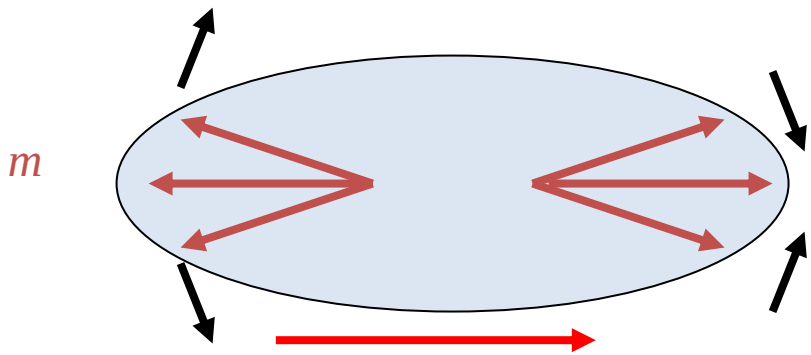


Favors an anti-parallel alignment of F to F1

Spin transfer torque (CPP geometry)

$$\frac{d\vec{m}}{dt} = \gamma_0 \vec{H}_{eff} \times \vec{m} + \alpha\, \vec{m} \times \frac{d\vec{m}}{dt} + \frac{1}{\tau}\vec{m} \times \left(\vec{m}_1 \times \vec{m}\right) + \frac{\xi}{\tau}\vec{m}_1 \times \vec{m}$$

Slonczewski (damping-like) field-like



destabilizing

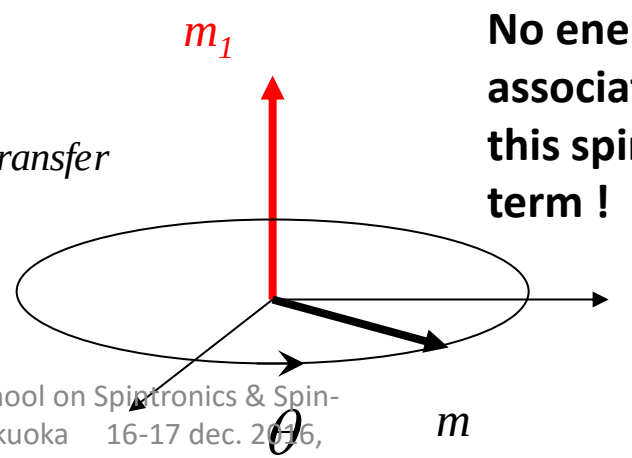
$$\left.\frac{d\vec{m}}{dt}\right)_{spin\ transfer}$$

stabilizing

$$\left.\frac{d\vec{m}}{dt}\right)_{spin\ transfer}$$

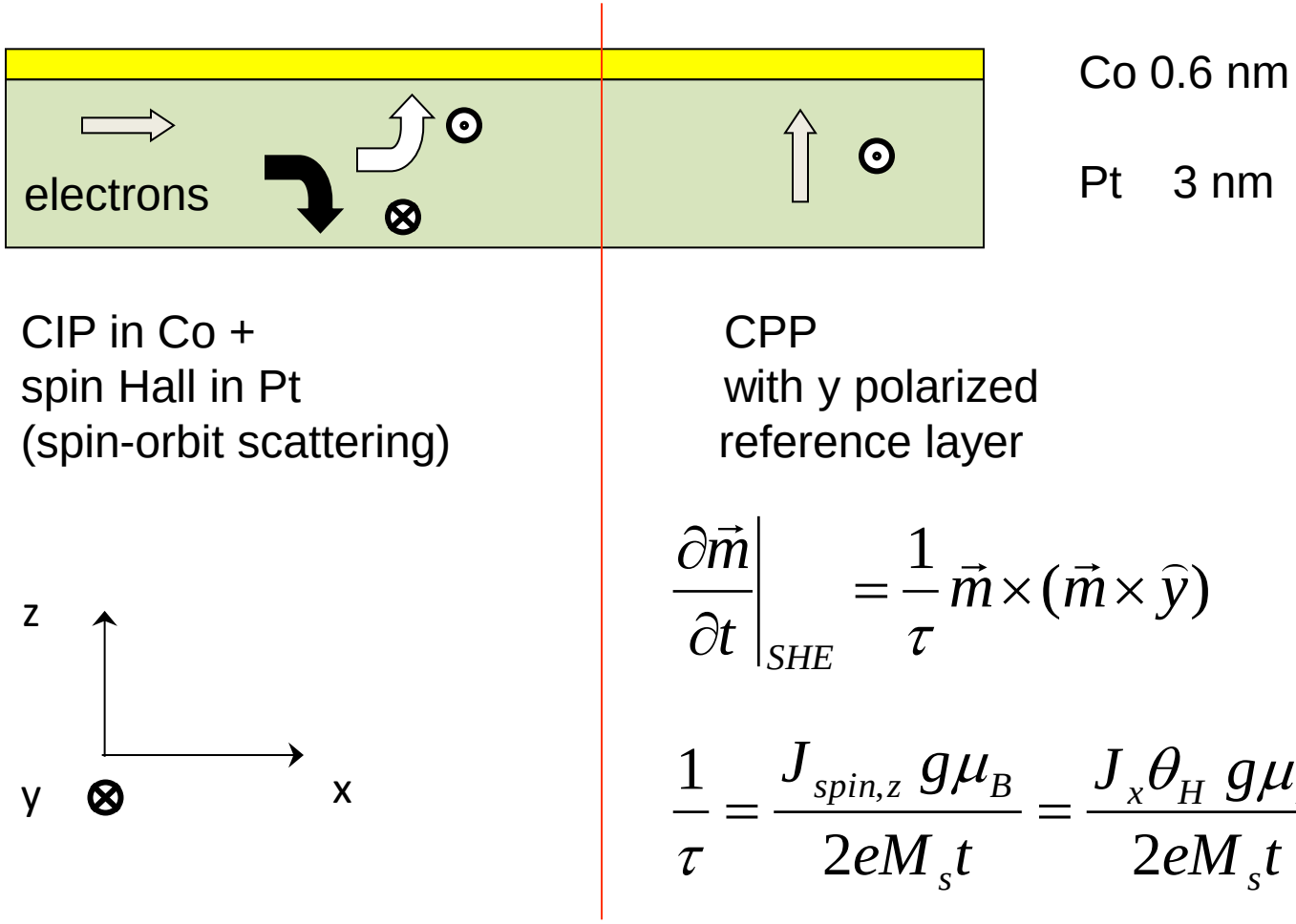
Effective field for the Slonczewski term

$$H_{\text{eff,spintransfer}} = \frac{1}{\gamma_0 \tau}\vec{m} \times \vec{m}_1$$



No energy term to be associated with this spin transfer torque term !

Torque by the spin Hall effect in an adjacent layer



$$\left.\frac{\partial \vec{m}}{\partial t}\right|_{SHE} = \frac{1}{\tau}\vec{m}\times(\vec{m}\times\hat{y})$$

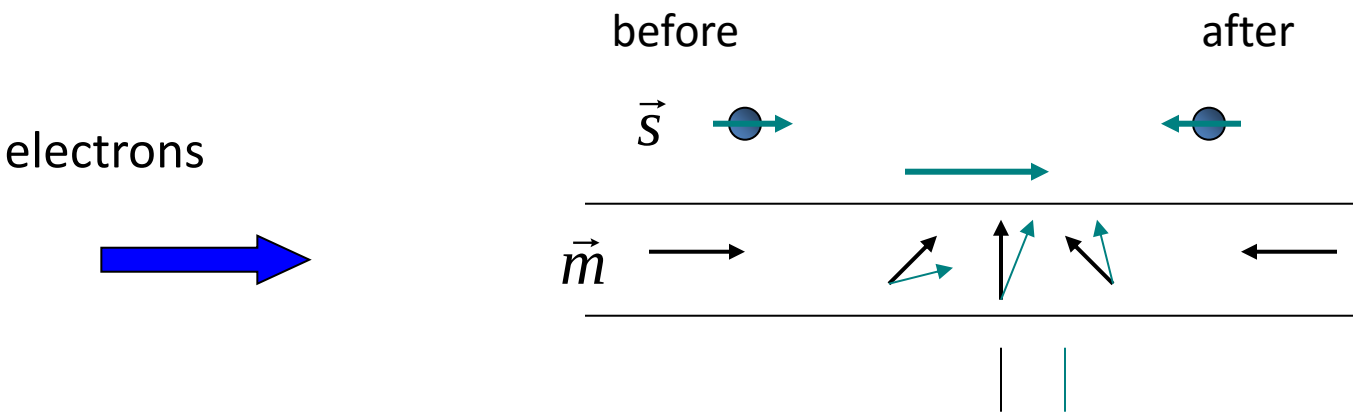
Slonczewski
CPP-STT

$$\frac{1}{\tau} = \frac{J_{spin,z}\, g\mu_B}{2eM_s t} = \frac{J_x \theta_H\, g\mu_B}{2eM_s t}$$

First demonstration of the effect on domain walls :
P.P.J. Haazen, E. Murè, J.H. Franken, R. Lavrijsen, H.J.M. Swagten, B. Koopmans
Nat. Mater. **12**, 299 (2013)

Spin transfer torque (CIP geometry)

L. Berger, J. Appl. Phys. **49,** 2156 (1978)



Adiabatic limit (walls are wide): carrier spins always along local magnetization

$$\frac{J}{e} P(\vec{s}(x) - \vec{s}(x+dx))$$

CPP spin transfer between successive *x* slices

-> angular momentum given per unit time in the slab *dx*

$$= \frac{J}{e} P \frac{\hbar}{2} \frac{\partial \vec{m}}{\partial x} dx \quad = \quad -\frac{M_s}{\gamma} \frac{d\vec{m}}{dt} dx$$

Spin transfer torque in continuous form (CIP)

$$\left.\frac{d\vec{m}}{dt}\right|_{spin\_transfer} = -u\frac{\partial \vec{m}}{\partial x} = -(\vec{u}\cdot\nabla)\,\vec{m}$$

« adiabatic » term

$$u = \frac{J\,P\,g\,\mu_B}{2e\,M_s}$$

u : a velocity that expresses the spin transfer (spin drift velocity) (Zhang & Li : b_J)

Permalloy : $\frac{g\,\mu_B}{2e\,M_s} = 7\,10^{-11}\,m^3/C$

1 x 10^{12} A/m^2 & P = 0.5 ⟷ u = 35 m/s

Full LLG equation under CIP-STT

$$\partial_t \vec{m} = \gamma_0 \vec{H}_{eff} \times \vec{m} + \alpha \vec{m} \times \partial_t \vec{m} - u \partial_x \vec{m} + \beta u \vec{m} \times \partial_x \vec{m}$$

"adiabatic term" "non-adiabatic term"

$$\vec{H}_{eff} = -\frac{1}{\mu_0 M_s} \frac{\delta E}{\delta \vec{m}}$$

effective field of other micromagnetic terms

Solved form

$$\partial_t \vec{m} = \frac{1}{1+\alpha^2} \Big[\gamma_0 \vec{H}_{eff} \times \vec{m} + \alpha \gamma_0 \vec{m} \times \left(\vec{H}_{eff} \times \vec{m} \right) - u(1+\alpha\beta) \partial_x \vec{m} + u(\beta - \alpha) \vec{m} \times \partial_x \vec{m} \Big]$$

Initial velocity for step current

$$V_0 = \frac{1+\alpha\beta}{1+\alpha^2} u$$

A. Thiaville et al., Europhys. Lett. **69** 990 (2005)

The “non-adiabatic” term : many models

1) True non-adiabaticity for narrow walls ?

Requires sub-nm thin domain walls

G. Tatara et al., Phys. Rep. **468**, 213 (2008)

J.Q. Xiao, A. Zangwill, M. Stiles
PRB **73**, 054428 (2006)

2) Spin accumulation and precession

S. Zhang and Z. Li, PRL **93**, 127204 (2004)

$$\beta = \frac{\tau_{sd}}{\tau_{sf}} \Big/ \left(1 + \left(\frac{\tau_{sd}}{\tau_{sf}}\right)^2\right)$$

$\beta \ll 1$ expected for 3d metals

3) Ab initio calculations : spin-orbit coupling for the carriers

I. Garate et al., PRB **79** 104416 (2009)

4) Non-local effects

A. Manchon et al., arXiv 1110.3487
D. Claudio Gonzalez et al., PRL **108** 227208 (2012)
C. Petitjean et al., PRL **109**, 117204 (2012)

Antisymmetric exchange in asymmetric structures (Dzyaloshinskii-Moriya interaction : DMI)

$$E_{ij} = \vec{S}_i \cdot \left(\overline{\overline{M_{ij}}}\, \vec{S}_j \right)$$

$$\overline{\overline{M_{ij}}} = \overline{\overline{Sym_{ij}}} + \overline{\overline{Antisym_{ij}}}$$

« pseudodipolar »
« anisotropic exchange »

$$\overline{\overline{Sym_{ij}}} = \begin{pmatrix} A_{ij}^1 & 0 & 0 \\ 0 & A_{ij}^2 & 0 \\ 0 & 0 & A_{ij}^3 \end{pmatrix}$$

(good base)

$$\overline{\overline{Antisym_{ij}}}\, \vec{S} = -\vec{D}_{ij} \times \vec{S}$$

$$E_{ij}^{antisym} = \vec{S}_i \cdot \left(\vec{S}_j \times \vec{D}_{ij} \right) = \vec{D}_{ij} \cdot \left(\vec{S}_i \times \vec{S}_j \right)$$

« antisymmetric »

Only if no inversion symmetry

The micromagnetic forms of DMI

$$E_{ij}^{antisym} = \vec{D}_{ij} \cdot (\vec{S}_i \times \vec{S}_j)$$

$$E_{ji}^{antisym} = \vec{D}_{ji} \cdot (\vec{S}_j \times \vec{S}_i) = -\vec{D}_{ji} \cdot (\vec{S}_i \times \vec{S}_j) \Rightarrow \vec{D}_{ji} = -\vec{D}_{ij}$$

What about the orientation of the DMI vector ?

Localized magnetism, single crystal : Moriya rules apply

T. Moriya, Phys. Rev. **120**, 91-98 (1960)

Isotropic « bulk case » $\vec{D}(\vec{u}) = D\,\vec{u}$ Chiral spin spirals favored

Isotropic « interface case » $\vec{D}(\vec{u}) = D\,\vec{z} \times \vec{u}$ Chiral spin cycloids favored

$$e^{DM} = D\left[m_x \frac{\partial m_z}{\partial x} - m_z \frac{\partial m_x}{\partial x} + m_y \frac{\partial m_z}{\partial y} - m_z \frac{\partial m_y}{\partial y} \right]$$

Physics without fully solving the LLG equation:

Thiele equation

Collective coordinates model for domain wall dynamics

Derivation of the Thiele equation (1)

LLG equation $$\partial_t \vec{m} = \gamma_0 \vec{H}_{eff} \times \vec{m} + \alpha \vec{m} \times \partial_t \vec{m}$$

`solved' form $$\vec{H}_{eff} = \{\vec{m} \times \partial_t \vec{m} + \alpha \partial_t \vec{m}\} / \gamma_0 + \lambda \vec{m}$$

ASSUME a magnetization structure in RIGID MOTION

$$\vec{m}(\vec{r}, t) = \vec{m}_0(\vec{r} - \vec{R}(t)) \qquad \partial_t \vec{m} = -\sum_j V_j \frac{\partial \vec{m}_0}{\partial x_j}$$

Force on the structure $$F_i = -\frac{dE}{dR_i} = \mu_0 M_s \int \vec{H}_{eff} \cdot \frac{\partial \vec{m}}{\partial R_i} = -\mu_0 M_s \int \vec{H}_{eff} \cdot \frac{\partial \vec{m}_0}{\partial x_i}$$

$$F_i = \frac{\mu_0 M_s}{\gamma_0} \sum_j V_j \int \left(\vec{m}_0 \times \frac{\partial \vec{m}_0}{\partial x_j} + \alpha \frac{\partial \vec{m}_0}{\partial x_j} \right) \cdot \frac{\partial \vec{m}_0}{\partial x_i}$$

Derivation of the Thiele equation (2)

$$F_i = -\sum_j \left[\frac{\mu_0 M_s}{\gamma_0} \int \left(\frac{\partial \vec{m}_0}{\partial x_i} \times \frac{\partial \vec{m}_0}{\partial x_j} \right) \cdot \vec{m}_0 \right] V_j + \sum_j \left[\alpha \frac{\mu_0 M_s}{\gamma_0} \int \frac{\partial \vec{m}_0}{\partial x_i} \cdot \frac{\partial \vec{m}_0}{\partial x_j} \right] V_j$$

$$\boxed{\vec{F}_{gyro} + \vec{F}_{dissip} + \vec{F} = \vec{0}}$$

Gyrotropic force

$$\vec{F}_g = \vec{G} \times \vec{V}$$

case of a film in x-y plane of thickness *h*

$$G_z = \frac{\mu_0 M_s}{\gamma_0} \int \left(\frac{\partial \vec{m}_0}{\partial x} \times \frac{\partial \vec{m}_0}{\partial y} \right) \cdot \vec{m}_0 dxdydz = \frac{\mu_0 M_s}{\gamma_0} h\, 4\pi N_{Sk}$$

Dissipation force

$$\vec{F}_\alpha = \alpha \overleftrightarrow{D} \vec{V}$$

$$D_{ij} = -\frac{\mu_0 M_s}{\gamma_0} \int \frac{\partial \vec{m}_0}{\partial x_i} \cdot \frac{\partial \vec{m}_0}{\partial x_j} dxdydz$$

A.A. Thiele, Phys. Rev. Lett. **30**, 230 (1973)

Applications of the Thiele equation

Simple wall (G_z=0)

$$\alpha \overleftrightarrow{D}\vec{V} + \vec{F} = \vec{0}$$

$$D_{xx} = -\frac{\mu_0 M_s}{\gamma_0}\int\left(\frac{\partial \vec{m}_0}{\partial x}\right)^2 dxdz = -\frac{\mu_0 M_s}{\gamma_0}\frac{2}{\Delta_T}h$$

$$F_x = 2\mu_0 M_s H h$$

(both per unit length)

Defines the Thiele domain wall width Δ_T

$$\boxed{V_x = \frac{\gamma_0 \Delta_T}{\alpha} H}$$

A.A. Thiele, J. Appl. Phys. **45**, 377 (1974)

Thiele equation under CIP STT

$$\vec{G}\times\left(\vec{V}-\vec{u}\right) + \overleftrightarrow{D}\left(\alpha\vec{V}-\beta\vec{u}\right) + \vec{F} = \vec{0}$$

Free structure (F=0), no gyrovector

$$\boxed{\vec{V} = \left(\beta/\alpha\right)\vec{u}}$$

A. Thiaville et al., Europhys. Lett. **69**, 990 (2005)

SHE longitudinal force on a magnetic structure

LLG with SHE

$$\dot{\vec{m}} = \gamma_0 \vec{H}_{eff} \times \vec{m} + \alpha\, \vec{m} \times \dot{\vec{m}} - \frac{1}{\tau} \vec{m} \times (\vec{m} \times \vec{p})$$

Solve for $\boldsymbol{H}_{\text{eff}}$
(Thiele procedure)

$$\vec{H}_{eff} = \frac{1}{\gamma_0} \left[\vec{m} \times \dot{\vec{m}} + \alpha\, \dot{\vec{m}} - \frac{1}{\tau} \vec{m} \times \vec{p} \right] + \lambda\, \vec{m}$$

The forces are

$$F_x = \frac{dE}{dX} = \frac{\mu_0 M_s}{\gamma_0} \int \vec{H}_{eff} \cdot \partial_x \vec{m}$$

SHE: for j//x one has p//y

$$F_x^{SHE} = -\frac{\mu_0 M_s}{\gamma_0 \tau} \int (\vec{m} \times \vec{p}) \cdot \partial_x \vec{m} = \frac{\mu_0 M_s}{\gamma_0 \tau} \int (\vec{m} \times \partial_x \vec{m}) \cdot \vec{p}$$

SHE force along J according to Néel chirality !

DMI energy density (interfacial DMI)

$$e^{DM} = D \left[m_x \frac{\partial m_z}{\partial x} - m_z \frac{\partial m_x}{\partial x} + m_y \frac{\partial m_z}{\partial y} - m_z \frac{\partial m_y}{\partial y} \right]$$

$$= D \left[-(\vec{m} \times \partial_x \vec{m})_y + (\vec{m} \times \partial_y \vec{m})_x \right]$$

Collective coordinates models of domain wall dynamics

Slonczewski equations for DW field dynamics in bubble materials (films with large PMA)

J.C. Slonczewski, Intern. J. Magnetism **2**, 85 (1972)

…

A.P. Malozemoff, J.C. Slonczewski, *Magnetic domain walls in bubble materials* (Academic Press, 1979)

1D field dynamics of a Bloch wall

N.L. Schryer, L.R. Walker, J. Appl. Phys. **45**, 5406 (1974)

1D DW dynamics in nanowires of soft magnetic materials

A. Thiaville et al., J. Magn. Magn. Mater. **242**, 1061 (2002)
D. Porter & M. Donahue, J. Appl. Phys. **95**, 6729 (2004)
A. Thiaville & Y. Nakatani, in *Spin Dynamics in Confined Magnetic Structures III* (Springer, 2006)

Field

A. Thiaville et al., Europhys. Lett. **69**, 990 (2005)
A. Thiaville & Y. Nakatani, in *"Nanomagnetism and Spintronics"*, (Elsevier, 2009, 2013)

STT

Construction of the model(s)

LLG in (θ, φ) variables (here with only conservative terms)

$$\begin{cases} \sin\theta\,\dot{\varphi} - \alpha\dot{\theta} = \dfrac{\gamma_0}{\mu_0 M_s}\dfrac{\partial E}{\partial \theta} \\ \dot{\theta} + \alpha\sin\theta\,\dot{\varphi} = -\dfrac{\gamma_0}{\mu_0 M_s \sin\theta}\dfrac{\partial E}{\partial \varphi} \end{cases}$$

Assumption for the structure (here, 1D assumption)

$$\theta(x,t) = 2\,\mathrm{Atan}\left[\exp\left(\frac{x - q(t)}{\Delta(t)}\right)\right] \quad \varphi(x,t) = \Phi(t)$$

$$\begin{cases} \dot{\Phi} + \alpha\dfrac{\dot{q}}{\Delta} = \gamma_0 H_{app} \\ \dfrac{\dot{q}}{\Delta} - \alpha\dot{\Phi} = \gamma_0 H_K \sin\Phi\cos\Phi \end{cases}$$

Case of DW driven by easy axis field

J.C. Slonczewski, Intern. J. Magnetism **2**, 85 (1972)

The Walker solution (1D)

Walker field

$$H_W = \alpha H_K / 2 = \alpha K / \mu_0 M_s$$

Explicit solution for angle Φ (t)
for field applied at t=0

$$\eta = \frac{H_W}{H_{app}}$$

$$\tan\Phi = \eta - \sqrt{\eta^2 - 1} / \tanh\left[\frac{\gamma H_{app}}{1+\alpha^2} t\sqrt{\eta^2 - 1} + \mathrm{Atanh}\left(\frac{\sqrt{\eta^2 - 1}}{\eta}\right)\right] \qquad \eta > 1 \quad \left(H_{app} < H_W\right)$$

$$\tan\Phi = \eta + \sqrt{1 - \eta^2}\,\tan\left[\frac{\gamma H_{app}}{1+\alpha^2} t\sqrt{1 - \eta^2} - \mathrm{Atan}\left(\frac{\eta}{\sqrt{1 - \eta^2}}\right)\right] \qquad \eta < 1 \quad \left(H_{app} > H_W\right)$$

N.L. Schryer, L.R. Walker, J. Appl. Phys. **45**, 5406 (1974)

Wall velocity $$v = \dot{q} = \frac{\gamma\Delta}{\alpha} H_{app} - \frac{\Delta}{\alpha}\dot{\Phi}$$

Velocity loss due to precession

Linear mobility

Precessional regime, constant Δ $\quad H_{app} > H_W$

$$< \dot{q} > = \frac{\gamma\Delta}{\alpha}\left[H_{app} - \frac{\sqrt{H_{app}^2 - H_W^2}}{1+\alpha^2} \right]$$

Limit case without transverse anisotropy

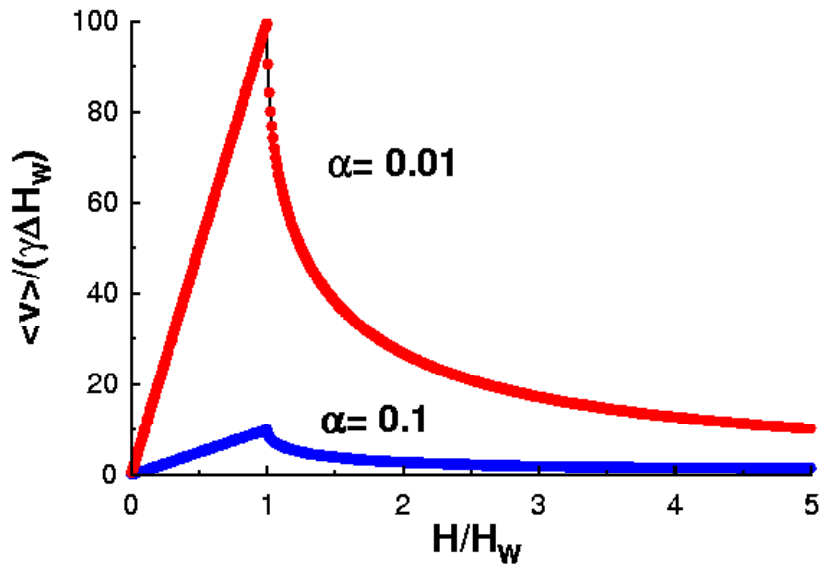
$$H_W = 0$$

$$\Phi = \frac{\gamma H_{app}}{1+\alpha^2} t$$

Uniform precession

$$q = \frac{\gamma\Delta\alpha}{1+\alpha^2} H_{app}\, t$$

« hard wall » mobility

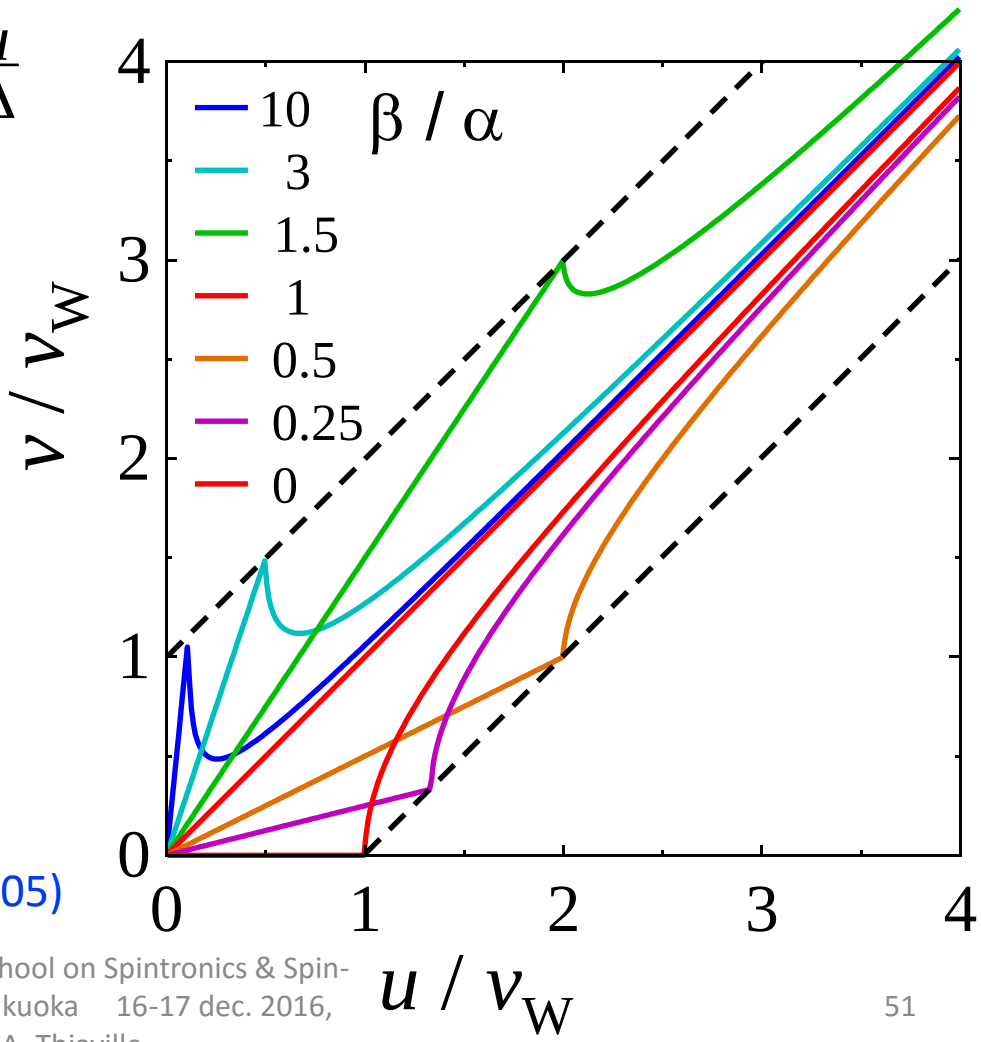


1D model in the case of STT

$$\dot{\Phi} + \alpha \frac{\dot{q}}{\Delta} = \beta \frac{u}{\Delta}$$

$$\frac{\dot{q}}{\Delta} - \alpha \dot{\Phi} = \gamma_0 H_K \sin \Phi \cos \Phi + \frac{u}{\Delta}$$

$$\Delta = C^{st}$$



Döring limit of stationary motion

$$|v - u| < v_W$$

For stationary motion

$$v = (\beta / \alpha) u$$

A. Thiaville et al., Europhys. Lett. **69**, 990 (2005)

What Micromagnetics does not (fully) describe: The Bloch point

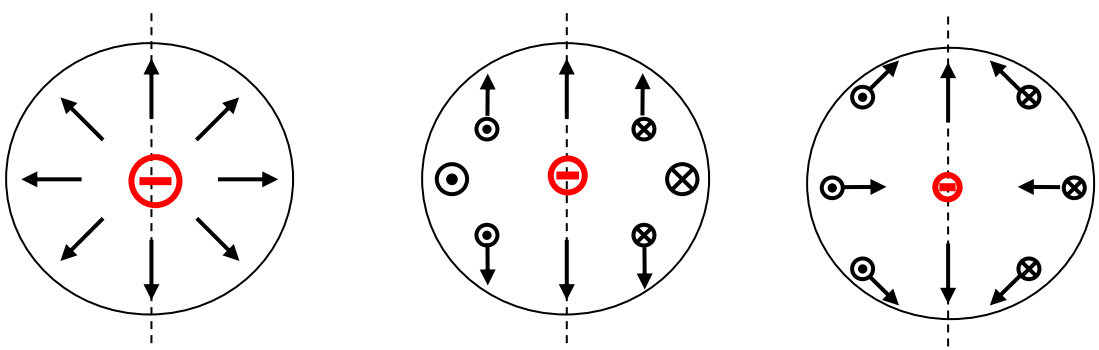
E. Feldtkeller, Z. angew. Phys. **19**, 530-536 (1965) [**17**, 121-130 (1964)]

Exchange

Energy density : $e_{exc} = \frac{2A}{r^2}$ for $\vec{m} = \frac{\vec{r}}{r}$ + rotations Diverges !

Total energy: $E_{exc} = 8\pi A R$ R radius where BP profile applies Finite !

Demag energy



hedgehog > circulating > spiraling

W. Döring, J. Appl. Phys. **39**, 57 (1968)

Conclusions & perspectives

Versatile micromagnetic framework

One (a few) phenomenological parameter(s) for each torque term

Atomic-scale Micromagnetics also exists

In most cases, a numerical calculation is required

But many physical insights can be obtained analytically

Limit: magnetization not fixed (ultrafast, large temperatures)

./..

References

A. Hubert, R. Schäfer, *Magnetic Domains* (Springer, 1998)

Handbook of Magnetism and Advanced Magnetic Materials, volume 2, H. Kronmüller and S. Parkin Eds. (Wiley, 2007)